SHANGHAI SPECIAL

Welcome to this special edition newsletter from the Central Maths Hub reflecting on the visit of our two Shanghai colleagues Vivienne and Echo. We thank Kings Norton Girls' School and John Henry Newman Catholic College (especially Elizabeth Bridgett and Kiran Bhangal) for hosting our Shanghai teachers and accommodating the vast numbers of you who have had the opportunity to observe the Shanghai teachers first hand.

Over 180 teachers have met and observed the Shanghai style lessons. Within this newsletter we will share some of the key findings and think about the implications for us as we continue to develop our collective understanding of Teaching for Mastery. We hope you enjoy reading this newsletter.

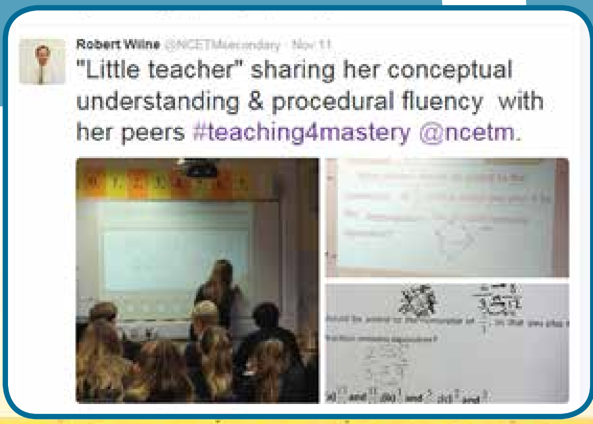
The Central Maths Hub Team

MEET THE SHANGHAI TEACHERS

Our two secondary school teachers from Shanghai arrived in the UK in November for a three week period. During this time they taught maths at Kings Norton Girls' School in Birmingham and John Henry Newman Catholic College in North Solihull to Year 7, 8 and 9 classes.

Ms Bian Xinyuan and Ms Sijun Wang (Echo and Vivienne) worked and taught alongside Elizabeth Bridgett and Kiran Bhangal, from Kings Norton Girls' School and John Henry Newman Catholic College, who went on a research visit to Shanghai in September.

The exchange is part of a project, funded by the Department for Education (DfE), to help English secondary school teachers understand and implement some of the key elements of Shanghai maths teaching that have proved so effective in helping school pupils in Shanghai reach exceptionally high levels of attainment. It follows on from the Primary Shanghai exchange that took place in September 2014 and February 2015.



TEACHING FOR MASTERY

"ALL THINGS ARE DIFFICULT BEFORE THEY ARE EASY."

The NCETM have published a discussion paper on Mastery which can be found at www.ncetm.org.uk/resources/45775

Some key points from the NCETM are distinguishing between:

A 'mastery' approach

A 'mastery' curriculum

'Mastery' of an area of mathematics

The NCETM provides the following guidance:

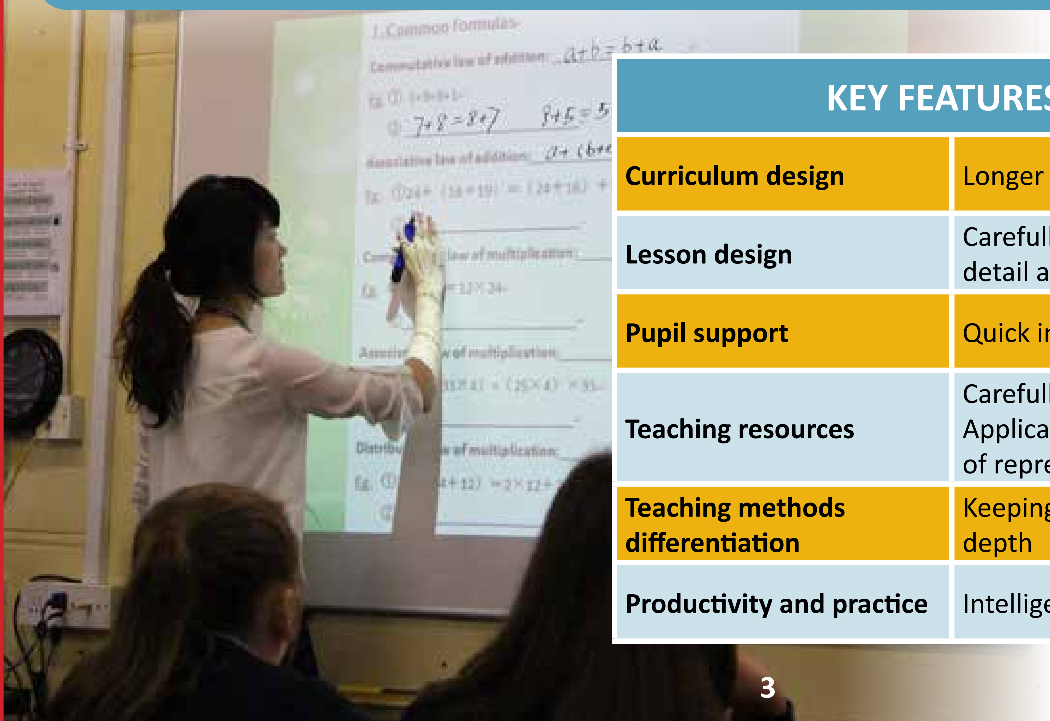
A 'mastery' approach; a set of principles and beliefs. This includes a belief that all pupils are capable of understanding and doing mathematics. Pupils are neither 'born with the maths gene' or 'just no good at maths'. With good teaching, appropriate resources, effort and a 'can do' attitude all children can achieve and enjoy mathematics.

A 'mastery' curriculum; one set of mathematical concepts and big ideas for all. All pupils need access to these concepts and ideas and to the rich connections between them. There is no such thing as 'special needs mathematics' or 'gifted and talented mathematics'. Mathematics is mathematics and the key ideas and building blocks are important for everyone.

'Mastery' of an area of mathematics; mastery is not just being able to memorise key facts and procedures and to answer test questions accurately and quickly. Mastery involves knowing why as well as knowing that and knowing how. It means being able to use one's knowledge appropriately, flexibly and creatively and to apply it in new and unfamiliar situations.

More recently, primary assessment materials have been produced to support teachers in making judgements about mastery and mastery with greater depth. These materials can be found on the NCETM website at the following link: **https://www.ncetm.org.uk/resources/46689**

The end of year 1 Maths Hub report on the England-China research and Innovation Project can be found here: **https://www.ncetm.org.uk/public/files/24603353/Shanghai+Exchange+Year+One+end-of-year+reports+August+2015.pdf**



KEY FEATURES OF MASTERY	
Curriculum design	Longer units of work, prioritising key topics
Lesson design	Carefully structured lesson to develop the detail and depth
Pupil support	Quick intervention
Teaching resources	Carefully chosen examples and activities. Application of variation theory. Effective use of representations.
Teaching methods differentiation	Keeping the class together and aiming for depth
Productivity and practice	Intelligent practice

THE SHANGHAI APPROACH

"A NATION'S TREASURE IS IN ITS SCHOLARS."

The National College's International Maths Research Programme (Phase Two) saw 50 SLEs travel to Shanghai to investigate approaches used in Chinese schools achieving top international rankings. The key findings highlighted five distinct, deeply rooted practices that define the Shanghai approach:

SYSTEM 1: PRACTICE AND CONSOLIDATION

Early training in number is the basis of all Maths learning with constant formal practice and repetition so that children demonstrate an assured fluency of use which supports accelerated progress. Mastery is achieved, not through reliance on repetitive drills, but through a rich variety in styles and approaches of practice questioning.

SYSTEM 2: SPECIALIST MATHS TEACHING

In order to qualify, teachers are required to have a degree in their specialist subject. Graduate Maths specialists teach primary Maths, whereas in England, primary Maths teachers are unlikely to have Maths beyond GCSE level.

SYSTEM 3: EFFICIENT TEACHING

Low class contact ratios mean the Chinese teach a small number of collaboratively planned lessons each day with a smaller spread of teaching groups so that some lessons are repeated with the same age classes. Teachers work together rather than in isolation.

SYSTEM 4 IMMEDIACY OF FEEDBACK AND INTERVENTIONS

Maths is taught in the morning: work is marked and returned by the end of the day. Prompt assessment supports rapid progress. Homework, handed in at the start of the day, is marked in time for the lesson later that day- a virtuous cycle of assessment supporting learning.

SYSTEM 5: PREVENTING RATHER THAN CLOSING THE GAP

Children are given additional help before they can fall behind, in the belief that everyone is capable of learning and that there are no intellectual boundaries to knowledge. Whereas in Shanghai, children work immeasurably harder than their teachers, in England the opposite is true.

(Source: NCTL Report on International Maths Research Programme China 2014)

LESSONS FROM THE LESSONS

"THE ANSWER IS ONLY THE BEGINNING"

Over the course of the exchange a number of observation sessions were arranged for teachers and other educators. Below are a number of extracts from some of these lessons. More information can be found in the Central Maths Hub online community.

www.ncetm.org.uk/community/13370

Each observation lasted for around an hour and was followed by a Teacher Research Group. This was a chance for all involved to identify and discuss the key features of the lesson and to analyse and evaluate pupil challenge, assessment and the resources used. The Shanghai teachers contributed to the discussions; including the analysis of their own delivery and areas for improvement.

FEEDBACK FROM TEACHERS:

Jessie Doyle
St Hubert's Catholic Primary School

"The very start of the lesson focused on homework that had been given out in the previous session. The homework was looking at being able to write and interpret an algebraic expression. It was apparent that a lot of time had been spent just focusing solely on this. Secondary school teachers have relayed that they would not necessarily spend as much time on just writing and interpreting algebraic expressions. From a primary perspective, with algebra being in year 6 now, this is possibly something primary teachers could invest time in, to set the foundations for secondary school. Homework was marked as a group and with every answer the children were asked to explain how they got their answer and encouraged to use mathematical terminology. When incorrect answers were given the teacher put it back to the rest of the class and asked if they agreed and if they didn't they were called up to the board and asked to model their answer. As a group we felt this build confidence in children because they were not been told their answer was wrong and were then been given the opportunity to correct their own work.

Throughout the rest of the lesson the children were still encouraged to use mathematical vocabulary, mainly 'distributive law' something the children struggled with and didn't seem to quite understand. The teacher also continued to ask children to keep explaining their answers, so she could observe their understanding of the concepts being taught, as a group we thought this was very powerful because it highlighted who was mastering the concept and who still needed support.

The main part of the lesson was looking at word problems and the children had to create an algebraic expression from the word problem, so children were having whole sessions focusing on one key concept. When speaking to the teachers after the session about this they said they teach the skills before the children actually get to solve a question, so in lesson 1 they focused on 'identifying equations', lesson 2 'the definition of an equation', lesson 3 'solutions to equations, is this correct?' and finally in lesson 4 'now solve an equation'. As a group we felt this is something that is missed in some teaching in England, but is vital to the children's understanding.

The group found the session really informative and primary and secondary school teachers said they already had some ideas to take back to their school.

OTHER COMMENTS FROM THE GROUP:

Deeper thinking questions reflect specialist maths teaching.

The teacher rarely modelled the method on the white board, they were drawn from the children who modelled methods on the white board themselves and discussed their thinking.

Lesson were at a slow pace, so children had time to really understand what was been taught.

All children were working on the same activity (there was no **obvious** differentiation).

Teaching took on a 'ping pong approach' so the teacher would stop every few minutes and draw children back to the question.

Language used such as 'judge' was simple but effective if getting the children to think critically.

A YEAR 8 LESSON ON EQUATIONS

The lesson began with the children standing behind their chairs until they were addressed and told to sit down. Question sheets for recording were given to the children as they entered the room. The sheets only contained a few carefully chosen problems.

The lesson began immediately with a problem displayed on the board.

There are 36 students in the class.
The number of boys is 6 more
than the number of girls.

A child was selected at random to read the problem to the class. The teacher (Echo) then asked how many girls there were in the class? Echo explained that she wanted the solution to the equation. She repeated this phrase. By this I believe she meant for the children to write the equation before solving it. She added;

Let x be the number of girls
find the number of boys

At this point she gave the children time to discuss the equation with their table partner. When asked for their input, the children immediately tried to respond with the answer so Echo made it clear that she wanted to find the solution to the equation by asking **'What is the equation?'.**

Echo told the children they needed to find the number of boys first and asked what the expression would be. She was given the answer x+6.

She then asked how she would write the equation to solve this problem. There was no response at first so she repeated the question a number of times until someone raised their hand to answer.

$2x + 6 = 36$

Echo explained the children do not need to simplify.

$x + (x+6) = 36$

The children were then told to find the solution, analyse the equation. At this point Echo told the children to substitute x for 15 to find the value of the left side of the equation. Echo wrote the solution:

$15 + (15+6) =$
$15 + 21 = 36$

She then highlighted to the children how the value of the left side was equal to the value of the right side. She emphasised the word equal when repeating her statement.

Echo explained x = 15 is the solution to the equation. She then went over each step again repeating the key vocabulary. She made it clear the children would be **checking** the solution to the equation not finding the solution to the equation.

She gave the same equation again and asked the children this time to substitute x for 14.

$x + (x+6) = 36$

The children were told to write down x=14 and asked to find the value of the left side (this vocabulary was repeated regularly throughout the session). The children were then given time to check the solution to the problem whilst Echo wrote a model of how to approach checking the solution on the board.

The value of the left side =
The value of the right side =
The value of the left side = The value of the right side.

The children were asked to explain how they checked the solution to the equation.

The value of the left side = 36
The value of the right side = 14 + (14+6) = 14 + 20 = 34
The value of the left side ≠ The value of the right side.

She put a strike through the = sign to show they are not equal. Echo repeated the finding that x = 14 is not the solution to the equation.

At this point Echo asked the children to look at example 1 on their sheets as she introduced a more complex equation. She used the word judge repeatedly. The example asked the children to judge whether -3 or -1 is the solution to the equation $4x^2 - 9 = 2x - 7$.

The children were told to substitute x for -3. She then asked them to give the equation for the value of the left side. The answer $4 \times -3^2 - 9 =$ was recorded. She asked the children if they agree to which there was little response so she prompted them to pay attention to -3^2.

Finally asking, do you think I need a bracket? And repeats the question before explaining to the children that they will get the wrong answer without a bracket. She then asked for the children to tell her the value of $(-3)^2$.

She continued by asking the child who answered to explain what $(-3)^2$ equals, prompting what does $(-3)^2$ mean? She wrote $(-3) \times (-3)$ on the board. Followed by $4 \times 9 – 9 = 36 – 9 = 27$ and highlighted the importance of paying attention to the bracket. The children continued to calculate the value of the right side. $2x – 7 = 2 \times (-3) – 7$.

An incorrect answer was given, followed by the correct answer and Echo asked the children to explain why the second answer was correct. This highlighted some weakness in the children's understanding of negative numbers. Echo briefly explained why the answer was correct by drawing a line and marking it with negative numbers.

Again the statements were repeated and the children were encouraged to calculate the values.

The value of the left side =
The value of the right side =
The value of the left side ≠ The value of the right side.
$X = -3$ is not the solution to the equation.

The children were asked to copy the model displayed on the board onto their own sheets before moving onto judging if x = -1 is the solution to the equation.

The value of the left side =
$4 \times (-1)^2 - 9 = 4 \times 1 - 9 = 4 - 9 = -5$
The value of the right side =
$2 \times (-1) - 7 = -2 - 7 = -9$
The value of the left side ≠ The value of the right side.

The children were asked is x = -1 the solution to the equation. Echo explained the process the children were undertaking was 'checking'. The children were told to record checking as the concept covered in the session.

Echo asked can we substitute x for (-3) directly? She continued on to explain we don't know if x = -3 is the solution so we must check if both sides are equal. She then asked the children to write this in the tip section of their sheet as their conclusion.

The children were then told to complete the exercises on the reverse of their sheet. She advised they should work quietly by themselves for 5 minutes and then could discuss with their table partner. Before letting them begin she pointed to the model on the board stating that the children should pay attention to it and that it was the format to use. Whilst the children worked independently Echo went around the room systematically checking the children's work and their understanding.

Finally Echo addressed the class and explained the focus of this lesson was on proving and being able to explain why it cannot be equal. She asked them what they had learnt today.

One child said 'The solution of the equation is equal and we have been checking the value of the right side equals the value of the left side'

Emma Barratt
Our Lady of Mount Carmel First School Academy

KEY FEATURES

- Questions are clearly designed in order to elicit misconceptions so that they can be addressed. The questioning used in the lesson provides a level of challenge to students.
- The Shanghai teachers use a centrally-produced reference book which outlines teaching methods and activities. This helps to avoid confusion for students, but also ensures that, for example, secondary school teachers are able to follow on from work studied at primary school.
- There is a clarity about what is important in the teaching, in terms of both what is being taught at the time and what will be important in the future.
- Value and time is given to the correct use of language, notation and the development of concepts (for example, the concept of equality was important in this lesson).

Dave Green
Shireland Collegiate Academy

KEY POINTS RAISED:

- The teacher was quick to respond if pupils were struggling.
- More challenging questions were introduced in the examples.
- It was clear to see the mastery of skills in the lesson progress but also how it fitted into previous lessons.
- Level of expectation.
- One lesson, one point.

Andrea Morgan,
Bishop Challoner Catholic College

A YEAR 7 LESSON ON SIMPLIFYING AND SUBSTITUTING INTO EXPRESSIONS

Context

The lesson observation took place at King Norton Girls School, led by a Shanghai teacher (Vivian). It was the final lesson in a two-week sequence of lessons with this mixed-ability year 7 class. In previous lessons, the students had looked at expressions: starting with the area and perimeter of a rectangle, looking at the associative and distributive laws, notation and simplifying.

To begin the lesson there were two questions on the board.

Q1

$8x + 7x$

$9a - 8a$

For each, the teacher asked:

"Who can tell me the answer?"

hands up- a student gave their answer

"Do you agree with her?"

whole class response ***"yes"***

"Can you tell me why?"

hands up again- a student gave their answer

For the second question, a student gave the answer "a" and when another student was answering why the teacher brought out the fact that it is 1a but we "omit 1".

This was then put on the board:

Q2

Note: When we simplify addition or subtraction we just add or subtract the numbers before the same letters.

Initially, the underlined words were blank and students were asked what goes there-they put their hands up to offer. Once the blanks were completed the whole class were asked to read out the note together.

These two questions were then put on the board.

$5 \times 7y$
$80b \div 2$

The answers were reviewed in a similar way to the other questions.

This was then put on the board:

Note: When we simplify multiplication or division you should do multiplication or division of the numbers before and then times the letters.

Again students were asked what goes where the underlined words are and then the whole class were asked to read out the note together.

The below was put onto the board:

Basic Training-

$9 \times 2x - 3x$

The teacher asked:

"When we meet multiplication and subtraction together, which do we do first?"

She said, ***"Tell me together."***

Students were then asked questions to complete the example and it was written on the board by the teacher.

Basic Training-

$9 \times 2x - 3x$
$= 18x - 3x$
$= 15x$

The teacher rubbed out the line "= 2 x 2a" and also pointed out that the rest of the working was the same as:

$= 12a + 4a$
$= 16a$

Then the below was put onto the board:

Variant Training-

$12a + (10a - 8a) \times 2$

The teacher asked:

"What should we do first?"

A student offered that the brackets should be done first, and the teacher wrote:

Variant Training-

$12a + (10a - 8a) \times 2$
$= 12a + 2a \times 2$

Students were then asked to do the rest of the question in their exercise book. The teacher said, ***"If you want to write on the whiteboard put your hand up."*** A student volunteered and was called up to write on the whiteboard. Whilst she did this, the teacher said, ***"If you have finished you can communicate with your classmate."***

The student finished writing on the whiteboard and the whole class were called to attention (always done by a clapped rhythm by the teacher which students clapped back).

Variant Training-

$12a + (10a - 8a) \times 2$
$= 12a + 2a \times 2$
$= 2 \times 2a$
$= 4a + 12a$
$= 16a$

The below was then put onto the board:

Simplify and find the value-

Pang went to buy some fruit. Per kilogram of apples costs 3 yuan. Pang bought x kilograms. How much did he spend?

Students were asked (hands up) and a student gave the answer, which was written on the board.

$3x$

The below was added to the board:

If Pang bought 2 kilograms. Pang spent (   ) yuan.

Students were asked what goes in the space (hands up). A student gave the answer ***"6".*** This wasn't written, but the teacher asked the follow up question, ***"Can you give me the expression?"***

A student gave the answer, which was written on the board.

$3 \times 2 = 6$

The below was added to the board:

If Pang bought 5 kilograms. Pang spent (   ) yuan.

The teacher asked, ***"x will be what?"***

A student gave the answer ***"5"*** and the below was written on the board:

$3 \times 5 = 15$

Then the below was put onto the board and students were asked:

Express this process with an algebraic expression:

Enter x

times 18	*18x*
add 32	*18x + 32*

Output

When x = 0 or 1 or 4 find the value of 18x + 32

x	0	1	4
18x + 32	32	50	

When completing x = 1, the students were guided through and shown the column method for adding 18 and 32 on the whiteboard.

When it came to x = 4, the teacher said that this one is ***"a little difficult to calculate in your brain"***

And wrote on the whiteboard:

Solution: when x = 4
18x + 32
= 18 x 4 + 32
= 72 + 32
= 104

The column method for multiplication of 18 and 4 was shown on the whiteboard.

The below was put onto the board:

Have a try: find the value

(1) When m = 8, find the value of 4m + 5

(2) When a = 3, b = 12, find the value of 9a – 2b

Students were reminded to use the same format as the above for writing their solution, and asked to answer the questions in their exercise book whilst the teacher circulated.

The teacher said, ***"If you finish, communicate with your classmate."***

A volunteer was asked to come up to the whiteboard to write out the solution. After the student had written their solution out, the teacher and the student talked through the solution together – with the student leading but the teacher pointing out the ***"first step"*** and ***"second step"*** and ***"third step"*** as the student explained. This was done for both questions.

Have a try: find the value

(1) When m = 8, find the value of 4m + 5
Solution: When m = 8
4m + 5
= 4 x 8 + 5
= 32 + 5
= 37

(2) When a = 3, b = 12, find the value of 9a – 2b
Solution: When a = 3, b = 12
9a – 2b
= (9 x 3) – (2 x 12)
= 27 - 24
= 3

At the end of the explanation for the second question, the teacher asked the class, ***"She adds brackets here, is this necessary?"*** (hands up). The student at the front was then told, ***"you can correct it now"*** and rubbed out the brackets.

The whole class were told, regarding their questions, ***"If you are not correct, you can correct it now."***

The below was put onto the board:

(3) When x = 17, find the value of 4x + 6x

The teacher said, ***"Please do in your exercise book – action."*** The teacher circulated and said, ***"If you have finished, you can communicate with your classmate."***

The teacher asked two children, in turn, to come up to the whiteboard to write up and explain their methods (students wrote them side by side on the whiteboard).

(4) When x = 17, find the value of 4x + 6x

Solution: When x = 17	*Solution: When x = 17*
4x + 6x	*4x + 6x*
= 4 x 17 + 6 x 17	*= 10x*
= 68 + 100	*= 10 x 17 = 170*
= 168	

During the student explanations the teacher brought out different aspects:

For the first method, the teacher asked the whole class, ***"Did she calculate it right?"*** and said, ***"Let's check it."***

She then modelled the column multiplication 17 x 6 and corrected the answer on the board. For the second method, the teacher pointed out regarding the ***"= 170", "Maybe I like to put = on the next line"***

The teacher then highlighted the fact that ***"There are two methods."*** and asked the whole class, ***"Which method is easier?"*** to obtain agreement from the class that the second method was easier.

The teacher said, ***"Let's summarise"***

This was then put on the board and the whole class were asked to read out together:

1. *Write "Solution" and the condition "when"*
2. *Write the algebraic expression*
3. *Simplify first if the algebraic expression can simplify*
4. *Put the number in it*
5. *Calculate the answer*

The below was put onto the board and students asked to try in their exercise book. Again, a student was asked to come up to the board to write their answer. They were then asked to explain their answer to the whole class, with the teacher highlighting the steps (writing 1, 2, 3, 4, 5 by the relevant lines of working as the student explained).

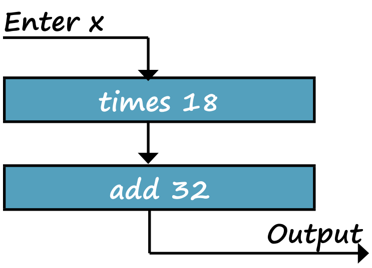
Simplify and find the value

When $x = 10$ find the value of $9x - 5x$

Solution: When $x = 10$

$$9x - 5x$$
$$= 4x$$
$$= 4 \times 10$$
$$= 40$$

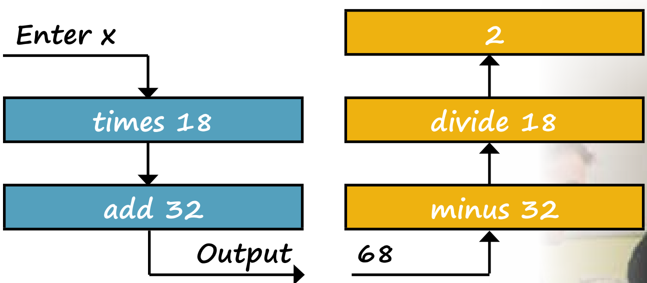
Then the below was put onto the board:



If I tell you the value is 68, x will be?

The teacher said ***"Communicate with your deskmate."***

The class then discussed the answer to this (led by teacher, with hands-up questioning).



If I tell you the value is 68, x will be?

This part was quite rushed as it was the end of the lesson, however, the class were still told that they must ***"check it is right or not"*** after obtaining the answer of 2, and modelled substitution of x = 2 into the original function machine to check that 68 was obtained.

LESSON STRUCTURE

The lesson was structured in small and clear learning steps, each building upon prior knowledge. Each step varied slightly from the one before - and there were no long periods of practice on each step, but instead each example/question used in the lesson was framed slightly differently.

EXPECTATIONS OF STUDENTS

All students were expected to achieve the same thing by the end of the lesson, regardless of the fact that the class was mixed ability. The class were kept together and all students working at the same pace. The 'small-steps' structure of the lesson, use of scaffolding (teacher or other students modelling and explaining the answer to **every question in the lesson** on the whiteboard, including modelling of steps in the questions that some students might find difficult such as column multiplication) and extension tasks ("if you have finished, communicate with your classmate why.") supported this.

STUDENT INDEPENDENCE

Although students were not working independently for much time in the lesson - they were actively involved in the learning through constant teacher whole-class questioning (of what and why), having a go at the questions on the board in their exercise book, being asked to explain why they had done what they had done to each other, some students modelling their answers on the whiteboard and exploration of mistakes or misconceptions that arise.

EMPHASIS ON PRECISION

This was evident both through the precise use of mathematical language by the teacher (and demanded of the students) and the precise way that mathematical solutions should be written down in exercise books – instructed as if it is part of the mathematical method, with deviations from this pointed out.

GENERALISATION

Generalisation happens after examples in the form of 'notes', which the whole class are asked to read out together. It is clear how the statements have arisen from the previous examples as the teacher had already highlighted the steps verbally during explanation of the precise examples.

Emma Penn
Tudor Grange Academy

A YEAR 7 LESSON ON SIMPLIFYING EXPRESSIONS

The lesson observation took place at Kings Norton Girls School Birmingham. The observation lasted for one hour and was followed by a Teacher Research Group for a further 30 minutes. The lesson was led by one of the Shanghai teachers with a number of colleagues observing the lesson from the back of the room. The Shanghai teachers had taught a couple of lessons to this group before and there was strong evidence that the lesson built on prior learning. The group was mixed ability (all girls) and this is an established arrangement in the school.

The lesson title was **"Simplifying 2".**

The lesson began with students being asked to review prior learning and complete the missing gaps (underlined) in the following 'note'

When we simplify addition or subtraction we just add or subtract numbers before the same letters.

The class were then made to chant this note aloud.

Next the students were asked to answer the following questions on mini-whiteboards.

$18x - 8x$
$5a + 2a$
$15m + 5m$
$18n - 18n$

Students coped well with the task and most (if not all) gave correct answers. The final example was drawn out in more detail. The teacher picked up two whiteboards, one read and the other . She then drew out that fact that and that when you multiply something by 0 the answer will be 0.

After this brief review the lesson now moved into a concrete example.

"Each exercise book costs yuan. Pang, Ding and Ya bought 3 respectively. How much did they spend altogether?"

The teacher emphasised the importance of the ***"3"*** and the word ***"respectively".***

Students were asked to solve the problem. As they worked the teacher circulated looking for interesting examples to draw out and use as key teaching points. Students were invited to come to the board and write down their solutions as the other worked. Initially two approaches were presented.

Method 1

$$3x + 3x + 3x = 9X$$

Method 2

$$3 \times 3 = 9$$
$$9 \times x = 9x$$

She then asked students if there were any other methods. A third student offered the following:

Method 3

$$3^2 = 9x$$

The teacher then facilitated a brilliant (at times painstaking, but nonetheless critical) discussion on each of these approaches. For method 1 she ensured that students *made the link* between each of the 3x terms and the problem posed. She made clear that the first of these terms was Pang, the second Ding and the third Ya.

For method 2 she questioned students on the different roles played by the two values of 3 in 3 x 3 = 9 (this is probably exactly why she chose this problem rather than 3 x 4). Students drew out the fact that the first three represented how many books were bought by each person and the second three represented how many people bought books (i.e. Pang, Ding, Ya) **all the time links were being made to the concrete problem posed so the mathematics was not abstract.** The teacher corrected the first line of method 2 to ensure it read 3 x 3x . The student who offered method 3 then quickly realised her method was similar to method 2.

Next the teacher asked the class to vote on which method was the easiest to use. The general response was method 1 "because it is addition". The teacher then asked, what if it was,

$$3x + 3x + 3x + 3x + 3x + 3x + 3x?$$

"How many 3x are here?"

Answer, ***"21"*** so the question was asked again. This time, "7".

The teacher stressed that this was a long calculation and a quicker way to do this was to calculate 3x x 7.

Students came to the realisation that method 2 would be easier in general as "it is shorter".

Even more time was now spent formally unpicking the algebraic structure of both methods.

Method 1	Method 2
$3x + 3x + 3x$ $= \boxed{(3 + 3 + 3)x}$ $= 9x$ *Distributive law of multiplication was highlighted.*	$3 \times 3x$ $= \boxed{3x\ (3 \times x)}$ $= \boxed{(3 \times 3)\ x}$ $= 9x$ *Associative law of multiplication was highlighted.*

In method 2 the teacher was at pains to stress the importance of "spreading" the 3x first before any subsequent multiplication. **The tiny steps were emphasised and not glossed over.** At this point I looked at the time and we were already 20 minutes into the lesson. I wondered, how quickly would I have moved on if I had been teaching this topic? Would I have explored these key steps in detail? Would I have stressed the importance of 'spreading' and the use of distributive and associative laws?

The lesson moved into "basic training". The first question posed was 3 x 4a . Again a student was chosen to present their answer as all worked on the problem. The solution presented was,

$$3 \times 4a$$
$$= 3 \times (4 \times a)$$
$$= (3 \times 4)a$$
$$= 12a$$

All steps were included no matter how seemingly small. The student also gave a highly impressive and articulate description of how she had approached the problem,

"First I spread the 4a to get 3 times the multiplication of 4 times a, then I used the multiplicative law of associativity then I multiplied 3 and 4 !!!!!"

The precise use of language and deep understanding of the multiplicative structure was clear to see.

The students then completed,

$$5b \times 8$$
$$= (5 \times b) \times 8$$
$$= (5 \times 8)b$$
$$= 40b$$

Again with clear steps explained.

Students then moved on to an activity which just required them to ***"give the answer directly".***

$$7a \times 6 =$$
$$5x \times 4 =$$

Each time the teacher asked, ***"Can you explain it?"***

After completing the activity students were asked to "give a note about it". This was a great example of moving from the specific to the general. The sense I got was that the notes form the set of general rules that can be applied - but were only written after the concept was explored in detail.

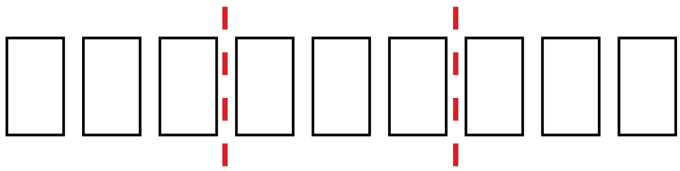
The lesson moved into another example.

"Ding, Pang, Ya spend 9x yuan altogether. How much did each person spend on average?"

Students quickly understood they had to do 9x ÷ 3. Another student presented the following method,

$$9x \div 3$$
$$= (9 \times x) \div 3$$
$$= (9 \div 3)\ x$$
$$= 3x\ (yuan)$$

Again they articulated the need to spread the   first, the fact that the order of operations can be interchanged and that the multiplication sign can be omitted. A visual demonstration was also given.

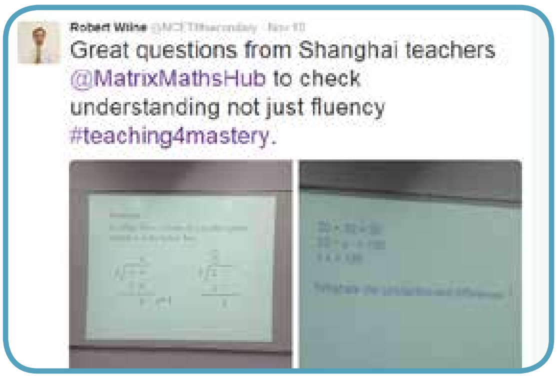


Students were given yet more "basic training" to complete independently.

As previously they were asked to answer the following problems "directly".

$$72a \div 9 =$$
$$25x \div 5 =$$

and then "give a note" about the process.



The lesson now started to move to more complex examples.

$$4 \times 4a + 6a$$

The teacher asked what the first step was and students recognised the need to calculate $4 \times 4a$. They arrived at the answer $22a$.

She then reformulated the question **to expose the potential misconception,**

$$6a + 4a \times 4$$

This time the reaction was mixed. A large proportion of the group thought the answer was $40a$. The teacher encouraged them to write another "note" about the order of operations.

The final activity was to **judge** whether the following statements were true or false using their arms.

$$10a - 3b + 2 = 7b + 2$$
$$3b + 7b \times 2 = 20b$$
$$10b + 60b \div 10 = 7b$$

TRUE

FALSE

At this stage the students were not all sure of the answers and this was probably a step too far for this lesson. Finally students were given a homework problem to complete.

KEY POINTS FROM THE LESSON

1. The tiny details are critical: Small steps were emphasised in all calculations.
2. The use of notes to move from specific to general: Notes were used to provide students with general results they could apply but only after the concept had been fully explored.
3. Misconceptions were explored: Problems were reformulated to ensure students had to think and address potential misconceptions.
4. The use of concrete problems: Algebraic structures were introduced in relation to a concrete and meaningful problem. Critically, that problem was used to explain each step of the abstract method, numbers and unknowns were connected to the problem.
5. Alternative methods: Showcasing a range of alternative methods and approaches.
6. Modelling: Solutions were presented in a clear manner. Students were only allowed to "give the answer directly" once they had a deep understanding of the approach

J Coughlan
The Central Maths Hub

MathsHUBS

QUOTES FROM TEACHERS

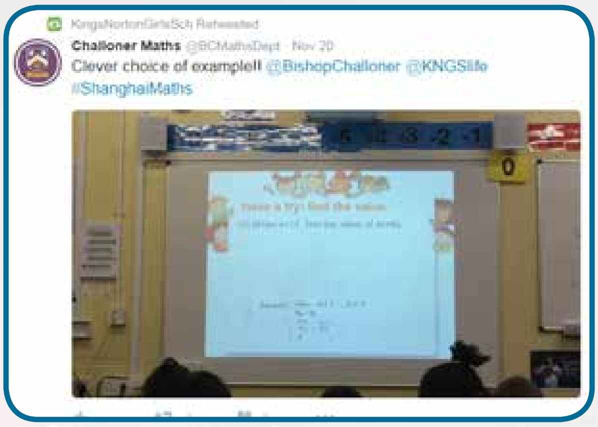
"TEACHERS OPEN THE DOOR. YOU ENTER BY YOURSELF."

"In the discussions following the lesson we spoke a lot about how the expectations in Shanghai are very different to here. Pupils are challenged and challenge is created, there is an expectation that pupils can access the new learning because they have been taught how to calculate quickly with numbers. There is a lot of emphasis on building pupil confidence to show solutions and accept mistakes. It is "normal for students to make mistakes"

Sarah Bellard, Smiths Wood Sports College

"The lessons tackle misconceptions head on. The teachers know what the common mistakes are going to be and stare them right in the eye. All the students in the class are expected to do chewy questions on each and every topic, no matter how simple on the surface. The students are given material that allows them to be fluent and confident in the language of maths. And after all of this, the teachers still meet every week to discuss how to improve their lessons, and where their students will have difficulties."

Elizabeth Bridgett, Kings Norton Girls' School



"The students own solutions were used - they were asked at two points in the lesson to write up their solutions and explain to the class. The teacher chose both correct and incorrect solutions drawing out the errors carefully by asking the class if they agreed with what was on the board. The teacher attended to specific aspects of the method of solution and drew the class back to this in each example e.g. attending to the 'changing signs' if there was a minus outside the bracket when expanding then attending to the multiplication."

Jayne Markie, Allen's Croft Primary School

"The approach showed the learning being broken down into tiny steps, which were continually repeated and reinforced throughout the lesson. This meant that 'teacher talk' was much greater than we in the UK might expect, and made the lesson appear far more teacher led. However, this also meant that students moved 'together' in their learning. Fewer questions may have been completed, but these were done really well, and in great depth. There was an emphasis on students sharing and explaining their thinking with 'Why?' being constantly asked throughout the lesson."

Nikki Jones, Shireland Collegiate Academy

"All students were expected to achieve the same thing by the end of the lesson, regardless of the fact that the class was mixed ability. The class were kept together and all students working at the same pace. The 'small-steps' structure of the lesson, use of scaffolding (teacher or other students modelling and explaining the answer to every question in the lesson on the whiteboard, including modelling of steps in the questions that some students might find difficult such as column multiplication) and extension tasks ("if you have finished, communicate with your classmate. Why?") supported this."

Emma Penn, Tudor Grange Academy, Solihull

HOW TO FIND OUT MORE ABOUT THE ENGLAND CHINA RESEARCH AND INNOVATION PROJECT?

"DO NOT FEAR GOING FORWARD SLOWLY; FEAR ONLY TO STAND STILL."

The end of this exchange by no means marks the end of the England-China project. The opportunity to observe the Shanghai teacher in action and the rich conversations had between teachers in the Teacher Research Groups will, hopefully, act as the springboard to continue these discussions further and develop a collective understanding of Teaching for Mastery that takes cognisance our own educational systems whilst being open to trying out new approaches.

EMAIL

If you have an idea as to how the lessons learnt from Shanghai can be further embedded within your own setting or you would like to try out some of the Shanghai methodologies, please let us know.

mathshub@bishopchalloner.bham.sch.uk

ONLINE COMMUNITY

You can continue the Shanghai discussions through the online community where links to resources, videos and other teachers' posts can be found.

www.ncetm.org.uk/community/13370

WEBSITE

The Maths Hub website also contains information on the England China project and will be updated regularly as the project moves.

www.mathshubs.org.uk/

TWITTER

Of course you can also always tweet us

@centralmathshub

The videos recorded of Elizabeth Bridgett speaking at the Chinese embassy have now all been posted on a video wall on the Maths Hubs programme website. http://www.mathshubs.org.uk/what-maths-hubs-are-doing/england-china/england-china-secondary/exchange-videos/